Deep Ocean Diving's Diving Science

Decompression theory - Bubble models

ABSTRACT
This page describes principles and theories about bubble generation and bubble growth in the scuba divers body and about the effect of bubble formation on decompression and decompression sickness (DCS, bends) in scuba diving. Whereas classical (neo-)Haldane theories are mainly empirical and only take dissolved gas into account, bubble theories intend to give a physical explanation of the effects of bubbles on decompression. Bubble theories take dissolved and free gas into account. Especially the Varying Permeability Model (VPM) and Reduced Gradient Bubble Model (RGBM) give good explanation.

History

In classic decompression theory according to Haldane and successors a certain amount of supersaturation of the divers tissue with dissolved inert gas is allowed. The divers tissue is divided in a number of hypothetical tissue compartments. A certain limit (M-value) is associated with each compartment to supersaturation levels of dissolved inert gas in the compartment (tissue tension). This theory suggests efficient decompression by pulling the diver as close to the surface as possible with constraint that in all tissue compartments the supersaturated tissue tension remains within the limits. By pulling the diver as close to the surface the pressure gradient between the supersaturated tissue tension and the pulmonary (or arterial) gas is maximized. This enhances the elimination of the excess gas in the tissue. This theory is mainly empirical and based on experiment. At the moment most diving tables and computers are based on this theory.
Since the early days, diving has become more sophisticated by diving deeper and longer, the use of other breathing mixtures, etc. Some tech divers have made their own adaptations to the decompression schedules by inserting depression stops at greater depth ('deep stops', sometimes called 'Pyle stops' after Richard Pyle). These divers report feeling better when using these deep stops. This suggests that classic decompression theory fails in some situations and cannot be extrapolated to every diving situation. In order to gain insight in the principles of decompression, forming of bubbles during decompression has been studied for the last three decades. This has resulted in new theories like the Varying Permeability Model (VPM) by Yount et al. and the Reduced Gradient Bubble Model (RGBM). Bubble theories do not only take into account the dissolved gas (like the Haldane models), but also the free gas in the divers body. In this chapter we will have a look at some features of bubble theory. Lots of mathematics will be presented. The most important equations however, will be highlighted.

Bubbles and surface tension

Consider a small air bubble in a glass of water. For the moment we neglect the solubility of the air in water. The small amount of air within the bubble is surrounded by a surface. The surface consists of water molecules which are unbound to one side. An unbound molecule represents more energy than a molecule which is completely surrounded by other water molecules. A *surface tension* γ is associated with this surface between air and water. The surface tension is the amount of energy per unit of surface area and is expressed in J/m^2 or N/m.
A system will always try to minimize energy. Surface tension tends to minimise the bubble's surface. Hence, a bubble tends to collapse. However, collapsing a bubble decreases its volume. This will increase the gas pressure in the bubble (Boyle's law), until equilibrium is established: the internal pressure compensates the surface tension. The internal pressure due to the ambient pressure and surface tension is given by the Laplace equation:

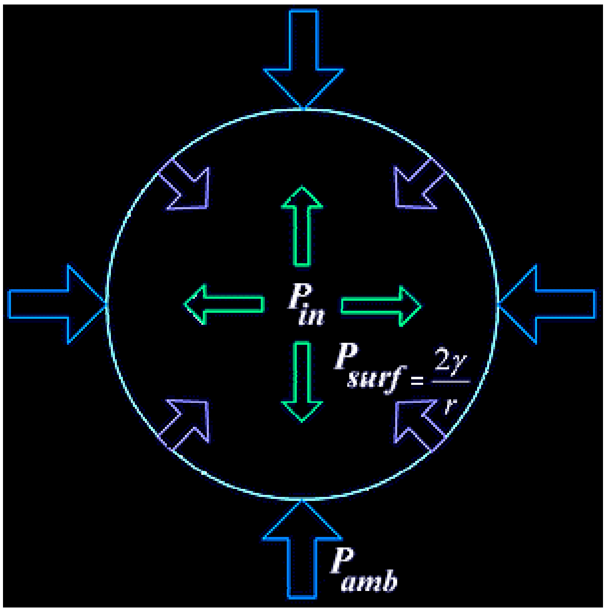

Figure 1: In equilibrium the internal pressure in the bubble is equal to the sum of the ambient pressure and the skin pressure due to the surface tension

$$P_{in} = P_{amb} + P_{surf} = P_{amb} + 2\gamma/r \qquad (1)$$

r Radius of the bubble in m

γ Surface tension in joule/m^2 of N/m. The surface tension of water at 273 K is 0.073 N/m.

P_{in} Pressure inside the bubble in N/m^2=10^{-5}bar

P_{amb} Ambient pressure in N/m^2=10^{-5}bar

P_{surf} Pressure due to the surface tension in N/m^2=10^{-5}bar

From this equation we learn that the smaller the bubble, the higher the pressure inside. You can experience the radius dependency of the pressure by trying to blow a balloon (bubble principles perfectly apply to a balloon up to the point where the balloon explodes). To get the first blow of air into the balloon (small radius) is a hell of a job, whereas it becomes easier if the balloon becomes larger.

Bubbles and diffusion

When we have a bottle of beer things get a bit more complicated (Usually the opposite holds, but when we look at the bubbles it might be). Bubbles in beer contain Carbon Dioxide. There is also Carbon Dioxide in solution in the beer. Carbon Dioxide can diffuse from the solution into the bubble or vice versa, depending on the partial pressure of the Carbon Dioxide in solution and in the bubble. If we assume that the bubble consist of only Carbon Dioxide, the Carbon Dioxide pressure in the bubble is given by equation (1) and depends on the radius of the bubble.

We define the partial pressure of the Carbon Dioxide in solution in the beer to be P_t. (If we regard the bottle of beer as a primitive model for a diver, we could call it 'tissue tension'). If the bottle is closed, the partial pressure of the Carbon Dioxide in solution P_t is in equilibrium with the ambient pressure P_{amb}. If we assume there is only Carbon Dioxide gas in the (closed) beer bottle, the beer is saturated with Carbon Dioxide and P_t will be equal to P_{amb} (we can neglect hydrostatic pressure). The pressure in the bubble P_{in} will be higher than P_t due to the surface tension. Gas from within the bubble will diffuse into solution and the bubble will collapse. So every bubble will collapse eventually due to this gradient P_{in}-P_t. This is why in a closed bottle of beer there are no bubbles and there is no foam. However, if we open the bottle things will be different. The ambient pressure will drop, whereas the value of P_t remains the same, at least for the moment. In this case P_t is larger than P_{amb}: the beer is supersaturated with Carbon Dioxide.

Given an ambient pressure P_{amb} and the partial pressure P_t of the Carbon Dioxide in solution, there is a critical bubble radius r^{min} at which the pressure inside the bubble P_{in} equals P_t. The critical radius can be found by substituting P_{in} by P_t in equation (1):

$$r^{min} = 2\gamma/(P_t - P_{amb}) \qquad (2)$$

For bubbles which size exceeds this critical size the pressure P_{in} in the bubble is smaller than the partial pressure P_t of the Carbon Dioxide in solution. Carbon Dioxide will diffuse from solution into the bubble. The bubble will grow. For bubbles smaller than the critical size, the opposite holds: gas from the bubble diffuses into solution and the bubble shrinks until it collapses completely. Bubbles at the critical size are in equilibrium, though it is an unstable equilibrium. This is depicted in Fig. 2.

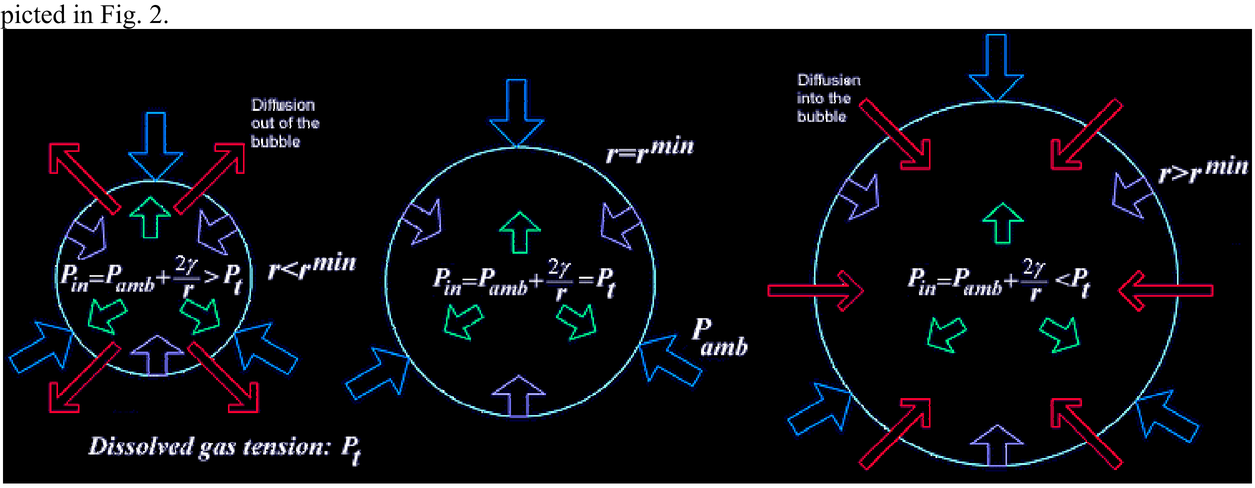

Figure 2:

So every bubble with a radius larger than r^{min} will start to grow. When we look at our opened bottle of beer we see bubbles becoming visible and heading for the surface, where they form foam. If you scrutinize a bubble you'll see that it grows during ascent. Its diameter might have doubled or tripled when it arrives at the surface. You might think this is due to Boyle's law . However it takes an ascent of several meters for a bubble to double its diameter. The growth of the bubble is due to the diffusion described above.

As an example, we can calculate critical radii for Spa Barisart Soda (6.4-8.0 g/l Carbon Dioxide). The pressure in the bottle specified by Spa is shown in next table (dependant on temperature). The partial pressure P_t of the Carbon Dioxide in solution is roughly that value. If we open the bottle the ambient pressure P_{amb} drops to 1 bar, whereas the partial pressure P_t remains at the high value. Using equation (2) we can calculate the critical radius r^{min}.

Temperature (°C)	**Pressure (bar=10^5 Pa)**	**r^{min} (µm)**
15	3	0.73
20	3.75	0.53
25	4.5	0.42
30	5.3	0.34
35	6	0.29
40	7	0.24

The Varying Permeability Model

According to previous chapter, in a supersaturated situation any bubble exceeding a critical size r^{min} will grow (and will disappear by floating to the surface) and any bubble smaller than this size will collapse. In a normal non-supersaturated situation, r^{min} approaches infinity. Any bubble will collapse. So we do not expect any bubbles around after a while. You might expect that if no initial bubbles are around, there is no bubble to grow on supersaturating the liquid. The tensile strength of water is estimated on 1000 atm, making immense supersaturations possible, before bubbles (voids) are created. If no initial bubbles would be present in the water making up the diver, a diver could easily dive to a kilometer depth and pop up to the surface without any problems. In practice, this is not the case. Bubbles form on modest decompression as low as 1 atm. Here comes in the *Varying Permeability Model* (*VPM*). The VPM was initially defined by Yount et al. [2] in order to give a quantitative explanation on the formation of bubbles in decompressed gelatin [1] (as model for divers tissue). Later on, they showed this model can be used to calculate dive tables as well [3], [4]. In next paragraphs we will have a look at the gelatin theory. Later on we will apply the theory to diving.

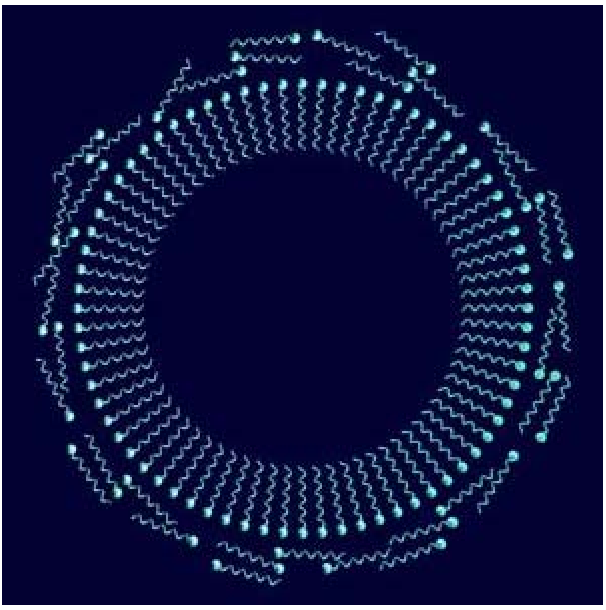

Figure 3: Skins of varying permeability are the base of the VPM

The gelatin experiments

Experiments on gelatin have been performed, by David Yount and other researchers [1]. The advantage of gelatin over water is that any bubble appearing during decompression gets trapped and won't flow to the surface. In this way they can be observed and counted. Yount applied the rudimentary pressure of Figure 4 to gelatin samples: Gelatin samples were made at ambient pressure $P_{amb}=P_0$ of 1 atm. The samples were rapidly compressed in a 100% Nitrogen atmosphere to $P_{amb}=P_m$. The samples were left at a pressure $P_{amb}=P_s=P_m$ for more than 5 hours. This period was long enough to fully saturate the sample at this pressure, so that $P_t=P_s$. After this, the samples were rapidly decompressed to a final pressure $P_{amb}=P_f$. After this decompression, bubbles formed in the sample. The number of bubbles were counted. Pressure changes are regarded fast: during the changes no gas is taken up or removed from any bubble.

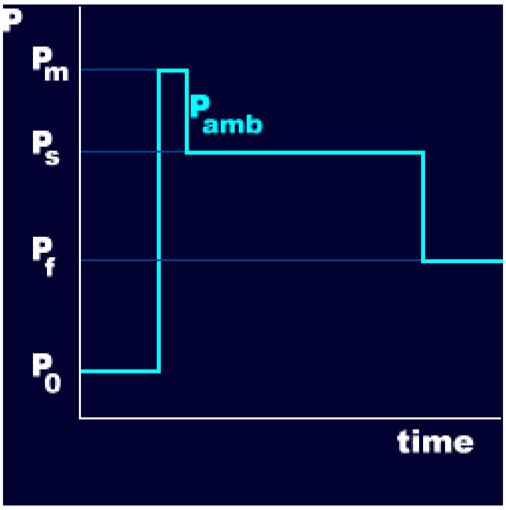

Figure 4: Rudimentary pressure schedule applied to the gelatin samples by Yount.

Basic concepts

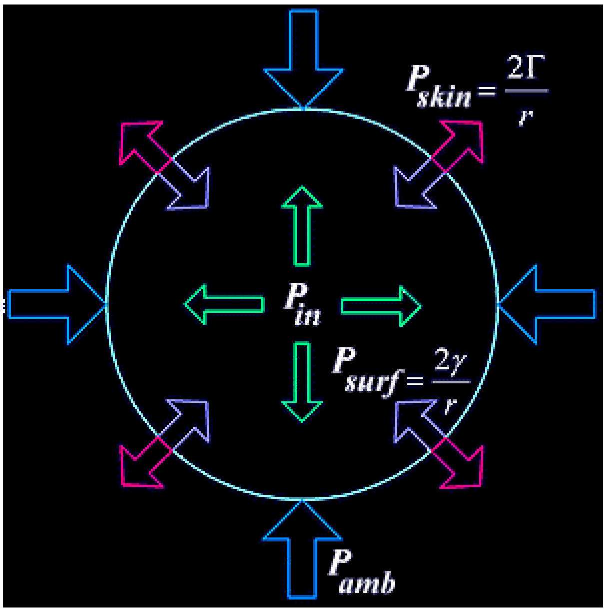

Figure 5: Pressures acting on the surface of the bubble.

According to the VPM, in aqeous media like water and gelatin stable gaseous cavities are present. They are called *nuclei*. Radii range from a few 1/100 µm up to around 1 µm. Any nucleus in water larger will flow to the surface and disappear. Whereas an ordinary bubble with these radii would collapse under normal conditions (no supersaturation), these nuclei appear to be exceptionally stable and have a long life. Yount proposed this stability is due to an elastic skin made up of surfactant, as shown schematically in Figure 3. Surfactant consists of (hydrophobic) surface active molecules, which are aligned. During the compression stage, these skins are permeable for gas up to a pressure of around 8 atm. Diffusion through the skin takes place. The pressure P_{in} of the gas in the nucleus is equal to the dissolved gas tension P_t in the surrounding liquid. Above this pressure, the skin becomes impermeable. Upon decompressing (reducing the ambient pressure) the skins are regarded permeable. The skin gives rise to a *'surface compression'* Γ which opposes the regular surface tension γ of the water/air surface, as shown in Figure 5:

$$P_{in} + 2\Gamma/r = P_{amb} + 2\gamma/r \tag{3}$$

The skin tension Γ is not constant but ranges from 0 to a maximum γ_c, which is called the 'crumbling compression'. The idea is that small variations of the size of the nucleus can be supported by varying the distance between the molecules in the skin. This gives rise to varying Γ. This situation is described by equation (3) and is referred to as the small-scale situation. In this equilibrium situation and in the permeable region, due to diffusion the internal pressure P_{in} is equal to the tension P_t. In the samples (no hydrostatic pressure, 100% Nitrogen) P_t equals P_{amb}. So $P_{in} = P_t = P_{amb}$. In this situation Γ equals γ, according to equation (3).

Upon compressing and decompressing, variation of the size of the nucleus becomes to large to be supported by varying distances between molecules. Surfactant molecules have to be expelled from or taken up into the skin in order to compensate for the area decrease resp. increase of the nucleus. This is schematically shown in Figure 6. The skin is surrounded by an amount of surfactant, which is not part of the skin. This amount acts as a reservoir, taking up or supplying surfactant molecules from or to the skin. The reservoir molecules are not aligned and cannot support a pressure gradient. Γ takes its crumbling value γ_c in this large-scale situation. Yount proposes two derivations of the VPM [2]: one from a thermodynamic point of view and one from a mechanical point of view.

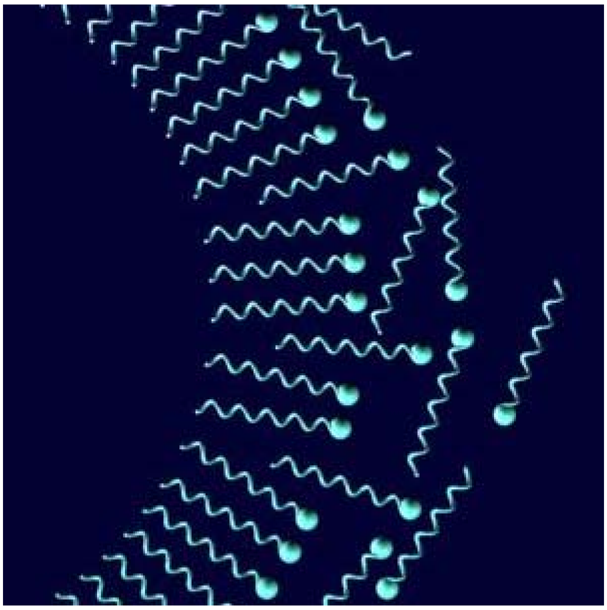

Figure 6: The large-scale situation: variation in the size of the nucleus result in expelling molecules from the skin

In the original sample there is a initial distribution of nuclei with radii distributed according to some function $f(r_0)$. (The '0' in r_0 refers to the *initial* situation). On applying the pressure schedule, it is assumed that all nuclei with a radius larger than some minimal initial radius r_0^{min} will grow into bubbles. The number of bubbles N that occur is given by the integration of $f(r_0)$ from r_0^{min} to infinity.

$$N = \int f(r_0)\, dr_0 \text{ , integration from } r_0^{min} \text{ to } \infty \tag{4}$$

Applying this theory to a diver, it might be assumed that the severity of Decompression Sickness (DCS) might be related to this number of bubbles, which occur after decompression. Hence, r_0^{min} becomes an indication for the severity of DCS. It is assumed that no nuclei are extinguished or created during application of the pressure schedule. Furthermore it is assumed that the ordering of nuclei is preserved: if one nucleus is larger than an other one, this is still true after a pressure change (ordering hypothesis). At the end of the pressure schedule there is a new distribution of radii $g(r_f)$ and a new radius r_f^{min} above which all nuclei will grow into bubbles. Note: a nucleus with radius r_0^{min} ends up as a nucleus with radius r_f^{min} after application of the presure schedule. The aim of next VPM calculations is

1. To define the allowed number of bubbles by defining r_0^{min}
2. To find a relation between r_f and r_0 (and hence between r_f^{min} and r_0^{min})
3. To find the relation based on r_f^{min} which governs the bubble formation on decompression.
4. To calculate the resulting restricting relations for the pressure schedule, given the value of r_0^{min} and hence, the number of resulting bubbles after application of the pressure schedule.

Thermodynamic equilibrium

From a thermodynamic point of view the left-hand side of equation (3) represents the skin pressure P_S:

$$P_S = P_{in} + 2\gamma_c/r \tag{5a}$$

Γ has been replaced by the large-scale value γ_c. Similarly the right hand term of equation (3) represents the reservoir pressure P_R:

$$P_R = P_{amb} + 2\gamma/r \tag{5b}$$

In the large-scale situation transport of surfactant is not described by setting P_R equal to P_S but by the requirement that the electrochemical potential in the skin and reservoir are equal. The electrochemical potential ξ is given by

$$\xi = \mu + kT\ln(\rho) + pv + Ze\psi \tag{6}$$

ξ	Electrochemical potential
μ	Pure chemical potential
k	Bolzmann constant
T	Absolute temperature in K
ρ	Molecular concentration or number density
p	Static pressure
v	Active volume occupied by one surfactant molecule
Ze	Effective charge of one surfactant molecule
ψ	Electrostatic potential

In the reservoir we have

$$\xi_R = \mu_R + kT\ln(\rho_R) + P_R v + (Ze\psi)_R \tag{7a}$$

and in the skin we have

$$\xi_S = \mu_S + kT\ln(\rho_S) + P_S v + (Ze\psi)_S \tag{7b}$$

Requiring ξ_R is equal to ξ_S and substituting P_S and P_R by the values in equation (5a) resp (5b) results in:

$$P_{in} + 2\gamma_c/r - \beta = P_{amb} + 2\gamma/r \tag{8a}$$

in which

$$\beta = [kT\ln(\rho_R/\rho_S) + (\mu_R - \mu_S) + (Ze\psi)_R - (Ze\psi)_S]/v \tag{8b}$$

Equations (8) can be used to calculate the changes in radii after applying a pressure step. We have a look what happens when applying the pressure schedule of Figure 4 to the sample. At the beginning of the pressure schedule $P_{amb}=P_0$. The pressure of the gas in the nucleus is

$$P_{in}=P_{t0}=P_{amb}=P_0 \tag{9}$$

Just before compression equation (8a) is:

$$P_0 + 2\gamma_c/r_0 - \beta_0 = P_0 + 2\gamma/r_0 \tag{10a}$$

After the pressure rise to the pressure P^* where the skins becomes impermeable equation (8a) reads:

$$P_0 + 2\gamma_c/r^* - \beta^* = P^* + 2\gamma/r^* \tag{10b}$$

Substracting equation (10b) from (10a), assuming $\beta_0=\beta^*$ and rewriting it a bit result in:

$$2(\gamma_c-\gamma)[(1/r^*)-(1/r_0)] = P^*-P_0 \tag{11}$$

We continue to compress rapidly from P^* to P_m. Since the skin is not permeable now, the pressure in the nucleus varies with its radius according to Boyle's law ($PV = P\ 4/3\ \pi\ r^3$ = constant):

$$P_{in} = P_t^*\ r^{*3}/r^3 \tag{12}$$

In this equation r^* is the radius of the nucleus at the beginning of the impermeable process. P_t^* is the corresponding dissolved gas tension, which is equal to P_0. So after the compression to P_m we have:

$$P_0\ (r^*/r_m)^3 + 2\gamma_c/r_m - \beta_m = P_m + 2\gamma/r_m \tag{13}$$

Assuming $\beta_m=\beta^*=\beta_0$, subtracting equation (13) from (10b) and rewriting a bit we end up with:

$$2(\gamma_c - \gamma)\ [(1/r_m) - (1/r^*)] = P_m - P^* + P_0\ [1 - (r^*/r_m)^3] \tag{14}$$

We now have relations (equation (11) and (14)) between the radius r_m of the nucleus after compressing and the radius r_0 prior to compression. The saturation phase ($P_{amb}=P_s$) that follows saturates the liquid so that finally the dissolved gas tension $P_{ts}=P_s$. We might expect that the radius of the nucleus increases to its original value (see the the note in the 'Mechanical Equilibrium' section). However, this has not been observed. So we assume

$$r_s=r_m \tag{15}$$

In fact the radius restores quite slowly, but for the moment equation (15) holds. Prior and after decompressing (which is fully permeable) equation (8a) reads:

$$P_s + 2\gamma_c/r_s - \beta_s = P_s + 2\gamma/r_s \tag{16a}$$

resp.

$$P_s + 2\gamma_c/r_f - \beta_f = P_f + 2\gamma/r_f \tag{16b}$$

Assuming $\beta_f=\beta_s$ (not equal to β_m, subtracting equation (16b) from (16a) and rewriting a bit result in:

$$2(\gamma_c-\gamma)[(1/r_f)-(1/r_s)] = P_f-P_s \tag{17}$$

So we now have a relation between all radii of the nucleus during the entire profile. A nucleus with radius r_0 ends up as a nucleus with radius r_f through a number of stages (r^*, r_m, r_s) defined by the relations (11), (14), (15) and (17).

We now define the criterion for bubble formation, which is given the Laplace equation:

$$P_{in}-P_{amb} = P_s - P_f \geq 2\ \gamma/r_f^{min} \tag{18a}$$

There is no reference to Γ or γ_c in this equation. We assume the skin of the nucleus to be permeable. So the skin does not restrict bubble formation: gas simply flows through the skin and forms a gas shell outside the skin. If the skin should not be permeable as has been proposed by others some tearing strenght or tearing tension $\Gamma=-\gamma_T$ is introduced. The bubble forming equation becomes:

$$P_{in}-P_{amb} = P_s - P_f \geq 2\ (\gamma + \gamma_T)/r_f^{min} \tag{18b}$$

By combining equation (11), (14), (15), (17) and (18a) we find the VPM equations.

For the ever-permeable region $P_m \leq P^*$:

$$P_{ss}^{min} = 2\gamma(\gamma_c-\gamma)/\ (r_0^{min}\ \gamma_c) + P_{crush}\ (\gamma/\gamma_c) \tag{19}$$

For the permeable-impermeable-permeable situation $P_m>P^*$ we find:

$$P_{ss}^{min} = 2\gamma(\gamma_c-\gamma)/\ (r_0^{min}\ \gamma_c) + [P_m - P_0(r^*/r_m)^3]\ (\gamma/\gamma_c) \tag{20}$$

In these equations we have the crushing pressure

$$P_{crush} = (P_{amb} - P_t)_{max} = P_m - P_0 \tag{21}$$

and the supersaturation pressure

$$P_{ss}^{min} = (P_t - P_{amb})_{max} = P_s - P_f \tag{22}$$

Equation (20) can be written as:

$$P_{ss}^{min} = 2\gamma(\gamma_c-\gamma)/\ (r_0^{min}\ \gamma_c) + (\gamma/\gamma_c)\ (P^* - P_0) + (\gamma/\gamma_c)\ (P_m - P^*)\ /\ [1 + (r^{\sim}/B)] \tag{23a}$$

The parameters $r^{\sim}$ and B are defined as:

$$r^{\sim} = r^*(r^*/r_m) \tag{23b}$$

$$B = 2(\gamma_c - \gamma)\ /\ \{\ P_0[(r^*/r_m) + 1 + (r_m/r^*)]\ \} \tag{23c}$$

Mechanical equilibrium

The other way the VPM is derived is by looking from a mechanical point of view. Changes in nuclear radius can be calculated by the equation proposed by Love, which reads (in the VPM form, [2]):

$$2(\Gamma - \gamma)(\partial r/r^2) = \partial P_{in} - \partial P_{amb} \tag{24}$$

In the 'permeable' region of the VPM, P_{in} remains constant and equal to P_t. Here ∂P_{in} is 0. For large-scale variations in the 'permeable' region of VPM equation (24) reads

$$2(\gamma_c - \gamma)(\partial r/r^2) = \partial P_{amb} \tag{25}$$

In the 'impermeable' region, P_{in} is given by equation (12). Differentiated it reads:

$$\partial P_{in} = -\ (3\ P_t^*\ r^{*\,3}/r^4)\ \partial r \tag{26}$$

For the large-scale variations in the impermeable region, equation (24) reads

$$[2(\gamma_c - \gamma) + 3\ P_t^*\ r^{*\,3}/r^2]\ (\partial r/r^2) = \partial P_{amb} \tag{27}$$

Together with the Laplace equation (18a) and the assumption of equation (15), equation (25) and (27) can be used (by integrating) to derive the VPM equations (19) and (20): equation (11) and (17) can be obtained by integrating equation (25), equation (14) can be obtained by integrating equation (27). The derivation is given in [2].

Note: During the compression phase ∂P_{in} is zero in equation (24). Assuming the pressure schedule takes place in the permeable region, integration of (24) from $P_{amb}=P_0$ to P_m results in:

$$2(\gamma_c - \gamma)\ (1/r_m - 1/r_0) = P_m - P_0 \tag{28}$$

During the saturation phase that follows ∂P_{amb} is zero in equation (24). Integration of (24) from $P_{in}=P_0$ to P_s results in:

$$2(\gamma_c - \gamma)\ (1/r_s - 1/r_m) = P_0 - P_s \tag{29}$$

Adding equation (29) to (28) results in $r_s=r_0$, assuming $P_m=P_s$ (in fact in the non-permeable situation or a situation in which $P_m \neq P_s$ we could derive the same result, though it takes some more derivation). This suggests that the nucleus is fully restored to the original size during saturation. The effect of the crushing is lost in this situation. However, this is in sharp disagreement with experiment. Further indication is given by special cases in which a pressure spike is present at the start of the schedule (Figure 4), so that $P_m<P_s$. In this cases the bubble count only depend on P_{ss} and P_{crush} and not on P_s! Hence, the assumption defined by equation (15) is made: $r_s=r_m$.

Equilibrium considerations

In this section we will consider some implications from the equilibria discussed above.

Rewriting equations (10a), (10b), (13), (16a) and (16b) gives us:

$$\begin{aligned} \beta_0 &= 2(\gamma_c - \gamma)\ /\ r_0 \\ \beta^* &= 2(\gamma_c - \gamma)\ /\ r^* - (P^* - P_0) \end{aligned} \tag{30}$$

$$\beta_m = 2(\gamma_c - \gamma) / r_m - [P_m - P_0 (r^*/r_m)^3]$$

$$\beta_s = 2(\gamma_c - \gamma) / r_s$$

$$\beta_f = 2(\gamma_c - \gamma) / r_f - (P_f - P_s)$$

(We recall our assumptions that β_0=β_m=β^* and β_s=β_f.) According to equation (8b) β is independent of radius. However, according to equation (30) β_0 appears to be a function of r_0. Assuming that β_0 is constant for all nuclei at P_{amb}=P_0 we obtain an remarkable prediction that γ_c increases with increasing r_0:

$$\gamma_c = \gamma + r_0\beta_0/2 \tag{31}$$

Another consideration stems from mechanical equilibrium: Small scale equilibrium is given by equation (3):

$$P_{in} + 2\Gamma/r = P_{amb} + 2\gamma/r \tag{32}$$

All properties of the skin and the reservoir are incorporated in the small scale skin compression Γ. The equation can be obtained from (8a) by setting:

$$2\Gamma/r = 2\gamma_c/r - \beta \tag{33}$$

Substituting the β values of equation (30) in (33) results at the respective ambient pressure values P_{amb}=P_0, P^*, P_m, P_s and P_f in:

$$\Gamma_0 = \gamma$$

$$\Gamma^* = \gamma_c - (\gamma_c - \gamma)\, r^*/r_0$$

$$\Gamma_m = \gamma_c - (\gamma_c - \gamma)\, r_m/r_0 \tag{34}$$

$$\Gamma_s = \gamma$$

$$\Gamma_f = \gamma_c - (\gamma_c - \gamma)\, r_f/r_s$$

A plausible small scale/mechanical equilibrium criterion for bubble formation is that Γ_f is less than or equal to zero. This results in:

$$2(\gamma_c-\gamma)/r_s \geq 2\gamma_c/r_f \tag{35}$$

Substituting this in equation (17) results in the Laplace equation (18a) as used for the thermodynamic derivation. Equation (34) shows that during the compression Γ increases. During saturation Γ 'relaxes' to its value prior to compression γ, keeping r_m constant. During decompression Γ drops to 0, the point at which bubble formation just starts.

Consequences of the VPM relations

Plotting P_{ss} vs. P_{crush}

Most conveniently, equation (19) is plotted as P_{ss}^{min} vs. P_{crush}. In these plots, P_{ss}^{min}-P_{crush} pairs resulting in the same number of bubbles (and hence, the same DCS morbidity) form straight lines.

Diver vs. gelatin

The VPM originally was developed to quantitatively explain bubble formation in gelatin during decompression [2]. The ultimate goal was to gain understanding of decompression sickness. To apply VPM to a diving situation it first was suggested that decompression sickness (DCS) symptoms were related to the number of bubbles. Say, severe symptoms occur at a number N_{DCS} of bubbles in some tissue. Given the radial distribution $f(r_0)$, equation (4) defines a r_0^{min}. If all nuclei with a radius equal or larger than this radius grow into bubbles, we end up with N_{DCS} bubbles (and some bad DCS). Given a dive to some depth resulting in an ambient gradient P_{crush}, equation (19) gives the maximum allowed gradient P_{ss}^{min} resulting in the N_{DCS} bubbles.

The VPM relations

The VPM relations (19) and (20) define the maximum allowed gradient between the ambient pressure and the tissue tension. In other words: it defines the minimum allowed ambient pressure P_{amb}, given the tissue tension P_t. In a diving situation it defines the depth the diver is allowed to ascend given the tissue tension. The relation should be applied to each tissue compartment of the diver.

The initial compression (defining P_{crush}) is important for P_{ss}^{min}. During this stage nuclei are crushed to a smaller size, making them less active in bubble formation. The secret lies in equation/assumption (15), which states that no regeneration of the bubble size takes place during saturation. It implies that a descent during a dive should be as quick as possible, the deepest part of the dive should be at the start of the dive and deeper dives should precede shallower dives in a repetitive dive situation. These facts have been empirically found during a century of decompression research.

Not a 100% Nitrogen saturation dive

The derivation of the VPM assumed 100% Nitrogen and fully saturated gelatin. If we apply the equations to a non-saturating diving situation in which the Nitrogen fraction is less than 100% (for example air, containing 79% Nitrogen), the VPM equations (19) and (20) can be regarded as a conservative restriction to the dive profile.

Applying VPM to diving

In this section we will apply the VPM to a diving situation and describe a method to generate diving tables. Whereas the VPM theory of previous sections applies to a special situation of fully saturated gelatin in a 100% Nitrogen atmosphere, situations during diving are different. The assumption that the severity of DCS is proportional to the absolute number of bubbles leads to very safe diving tables, not covering all of the conditions of modern diving tables and often leading to unacceptable long decompression periods. The VPM was reformulated, as described in this section, to fit it with conventional diving tables. Conventional diving tables were regarded as valid measurements. We will follow the derivation of Yount [3]. During this derivation we assume only one inert gas. Later on we will place remarks on using more inert gasses (Trimix, etc). Another assumption is the dive takes place in the 'permeable' region of the VPM.

The reformulated VPM

The derivation of the theory below is based on a number of more or less ad hoc assumptions. The most important assumptions concern the relationship between decompression symptoms and the amount of free gas (bubbles) in the divers tissue:

- There is an amount of bubbles N_{safe} which can be tolerated by the divers body, independent of all circumstances (like tissue tension, degree of supersaturation, etc). The initial critical radius corresponding to this number is r_0^{min} (equation (4)).
- The actual number of bubbles N_{actual} may be higher than N_{safe} as long as the total volume V of all free gas *always* remains below a critical value V_{crit}. This is called the 'critical-volume hypothesis'. A initial radius r_0^{new} smaller than r_0^{min} is associated with this number.
- The volume of free gas V inflates at a rate proportional to P_{ss} (N_{actual} - N_{safe}), where P_{ss} is the saturation P_t-P_{amb}.

The first of these assumption agrees with physiological studies, which state that the lungs are able to continue functioning as a trap for venous bubbles to a certain degree. From this assumption can be deduced that the rate at which the body can dissipate free gas by exchange in the lungs is proportional to both the supersaturation pressure P_{ss} and N_{safe}.
The assumption defined by equation (15) is fine tuned according to observations: the radius r_m slowly regenerates during saturation instead of remaining unchanged, as stated by equation (15). The regeneration is exponential, governed by a regeneration time constant τ_R:

$$r_s(t_R) = r_m + (r_0 - r_m)(1 - \exp(-t_R/\tau_R)) \qquad (36)$$

r_s Nuclear radius just prior to ascent and decompression (m)

r_m Nuclear radius after compression by P_{crush} (m)

r_0 Nuclear radius before descent (m)

t_R Regeneration period: time from start of dive up to start of ascent and decompression (min)

τ_R Regeneration time constant (min)

If we wait long enough the crushed nucleus will end up with its initial radius prior to compression r_0.
In contrast with other decompression models, VPM takes the effect of other gasses (water vapor, Oxygen, Carbon Dioxide) into account in calculating the tissue tension:

$$P_{t\_total} = P_{inert\_gasses} + P_{other\_gasses} \qquad (37)$$

P_{t_total} Tissue tension

P_{inert_gasses} Sum of the partial pressures of the dissolved inert gasses

P_{other_gasses} Pressure due to water vapor, Oxygen and Carbon Dioxide. Yount specifies a nearly constant value of 102 mm Hg (corresponding to 0.136 bar) for inspired partial Oxygen pressures up to 2 atm [5]

The supersaturation is now defined as:

$$P_{ss} = P_t - P_{amb} \qquad (38)$$

The reformulated VPM now consist of the following steps:

1. Specify the parameters defining the VPM: surface tension γ, the crumbling compression γ_c, the minimum initial radius r_0^{min}, the regeneration time constant τ_R and a composite parameter λ. The latter is related to the critical volume V_{crit}. The parameters are the same for each compartment.
2. Calculate the initial allowed supersaturation that is just sufficient to probe r_0^{min} and that results in N_{safe} bubbles. The equation for this is:
   $$P_{ss}^{min} = 2(\gamma/\gamma_c)(\gamma_c - \gamma)/r_s(t_R) \qquad (39)$$
   In fact, this is an enhanced equation (19), taking nuclear regeneration into account. In this equation the regenerated radius $r_s(t_R)$ is given by equation (36). Since the VPM parameters are the same for each tissue compartments, this initial allowed supersaturation gradient will be the same for each compartment.

3. Calculate a decompression profile, using this P_{ss}^{min}. The total decompression time defined by the profile is t_D.
4. Calculate a new allowed supersaturation gradient P_{ss}^{new} using:

   $$P_{ss}^{new} = 1/2\,[b + (b^2 - 4\,c)^{1/2}]$$

   where

   $$b = P_{ss}^{min} + \lambda\,\gamma / [\gamma_c\,(t_D + 1/k)] \qquad c = (\gamma/\gamma_c)^2\,\lambda\,P_{crush} / (t_D + 1/k) \tag{40}$$

   In these equation is $k=\ln(2)/\tau$, where τ is the half-time of the tissue compartment. This result in a larger allowed supersaturation gradient P_{ss}^{new}. Of course, this step is repeated for each tissue compartment.
5. Perform a number of iteration of step 3-4, until t_D and P_{ss}^{new} converge. Of course, occurrences of P_{ss}^{min} are now substituted by P_{ss}^{new}.

After the iterations we end up with a more severe decompression profile and a P_{ss}^{new} corresponding to a new initial critical radius r_0^{new}, which is smaller than r_0^{min}. This new radius results in a larger number of bubbles N_{actual} and a maximum volume of free gas approaching V_{crit}.

More inert gasses

In some (tech) diving situations, other gas mixtures are used consisting of more than one inert gas (for example Trimix, containing Oxygen, Nitrogen and Helium). In next remarks we assume Helium and Nitrogen to be the inert gases.

1. For each gas, the VPM parameters should be specified. For each tissue compartment a half-time for each gas should be specified.
2. For each gas, the allowed supersaturation gradient should be calculated using the method in previous section. In this case the supersaturation gradient for Helium is P_{ss_He} and for Nitrogen is P_{ss_N}
3. If P_{t_He} and P_{t_N} are the Helium and Nitrogen tissue tensions, the total tissue tension is given by:

   $$P_{t\_total} = P_{t\_He} + P_{t\_N} + P_{other\_gasses} \tag{41}$$
4. The allowed supersaturation gradient is given by the weighted average:

   $$P_{ss\_total} = P_{t\_total} - P_{amb} = (P_{t\_He}\,P_{ss\_He} + P_{t\_N}\,P_{ss\_N}) / (P_{t\_He} + P_{t\_N}) \tag{42}$$

Derivation

In this section we will derive the new VPM equation (40). The allowed supersaturation gradient P_{ss}^{min} as given by equation (19), (20) and (39) can be applied to diving as a safe-ascent criterion. Whereas they can be derived directly from VPM, the derivation of P_{ss}^{new} in equation (40) involves a number of ad hoc assumptions.

Assumption 1: The total volume of free gas in the divers body should never exceed a critical volume value V_{crit} at any time t (not during the dive, nor thereafter).

Assumption 2: The rate at which the free gas inflates is proportional to $P_{ss}(t)(N_{actual} - N_{safe})$. In this equation $P_{ss}(t) = P_t(t) - P_{amb}(t)$.

Assumption 1 and 2 result in the decompression criterion:

$$\int_0^t P_{ss}(t)\,(N_{actual} - N_{safe})\,dt \le \alpha V_{crit} \tag{43}$$

In this equation is α a proportionality constant. This criterion should hold for any t. To minimise the decompression time t_D, the $\le$ sign is replaced by the = sign.

Assumption 3: The actual number of bubbles N_{actual} and the number of bubbles always allowed N_{safe} are determined by the initial decompression stop and remain constant thereafter. The decompression criterion now reads:

$$\alpha V_{crit} = (N_{actual} - N_{safe}) \int_0^{t_{max}} P_{ss}(t)\,dt \tag{44}$$

In this equation t_{max} is the value of t at which the integral reaches the maximum value.

Assumption 4: The decompression profile is chosen so that $P_{ss}(t)$ remains at constant value P_{ss}^{new} during the ascent period t_D and decays exponentially to zero thereafter (at the surface). This is in agreement with Assumption 3: $P_{ss}(t)$ is always positive and never exceeds its initial value P_{ss}^{new}. This initial value is the maximum value defining N_{actual}. The latter remains constant thereafter. The exponential decay to zero is a conservative approximation: according to Yount&Lally [5] humans are 'inherently unsaturated' when equilibrated at atmospheric pressure by about 54 mm Hg (0.072 bar). Eventually, $P_{ss}(t)$ will become negative by this amount.

Due to Assumption 4 and the exponential decay, the integral of equation (44) reaches it maximum value in the limit as t_{max} approaches ∞. The criterion for decompression now becomes:

$$\alpha V_{crit} = (N_{actual} - N_{safe})\,\Big[\int_0^{t_D} P_{ss}^{new}\,dt + \int_{t_D}^{\infty} P_{ss}^{new}\,e^{-k(t-t_D)}\Big] \qquad \alpha V_{crit} = (N_{actual} - N_{safe})\,P_{ss}^{new}\,(t_D + 1/k) \tag{45}$$

In this equation is $k=\ln(2)/\tau$ where τ is the tissue compartment halftime.

Assumption 5: The distribution of nuclei in humans is not known. An decaying exponential relation is assumed, observed in vitro:

$$N_{actual} = N_0\,\exp(-\beta_0\,S\,r_0^{new} / 2kT) \qquad N_{safe} = N_0\,\exp(-\beta_0\,S\,r_0^{min} / 2kT) \tag{46}$$

β_0

VPM constant N_0

Normalization constant S

Constant area, occupied by one surfactant molecule in situ *k*

Boltzmann constant T

Absolute temperature

The decompression criterion can be rewritten:

$$P_{ss}^{new} = \alpha V_{crit} / [(N_{actual} - N_{safe})(t_D + 1/k)] \tag{47}$$

where:

$$(N_{actual} - N_{safe}) = N_0 [\exp(-\beta_0 S r_0^{new} / 2kT) - \exp(-\beta_0 S r_0^{min} / 2kT)] \tag{48}$$

Assumption 6: The exponential arguments in equation are small enough so that they can be expanded. According to [3] this approximation is in some question, since the model parameters are not fixed nor well known. The true distribution is unknown. According to this assumption equation (48) becomes:

$$(N_{actual} - N_{safe}) \approx (N_0 \beta_0 S r_0^{min} / 2kT)(1 - r_0^{new}/r_0^{min}) \tag{49}$$

Substituting (49) in (47) and rewriting this a bit results in:

$$N_0\beta_0 S/2kT\, r_0^{min} (1/r_0^{new} - 1/r_0^{min})(t_D + 1/k) P_{ss}^{new} - \alpha V_{crit} (1/r_0^{new}) = 0 \tag{50}$$

The radii r_0^{new} and r_0^{min} can now be replaced using the VPM equations (rewriting (30) and (19)):

$$\begin{aligned} r_0^{min} &= 2(\gamma_c - \gamma) / \beta_0 \\ 1/r_0^{min} &= \gamma_c [P_{ss}^{min} - P_{crush}(\gamma/\gamma_c)] / [2\gamma(\gamma_c - \gamma)] \\ 1/r_0^{new} &= \gamma_c [P_{ss}^{new} - P_{crush}(\gamma/\gamma_c)] / [2\gamma(\gamma_c - \gamma)] \end{aligned} \tag{51}$$

P_{crush} is here P_m-P_0. The equations (51) apply to the permeable region. Applying them to the impermeable region results in an acceptable error of only 3% for values of P_{crush} below 10 bar. Substituting r_0^{min}, $1/r_0^{min}$ and $1/r_0^{new}$ in the relations (50) by the equations given in (51) results in:

$$N_0 (\gamma_c - \gamma) S/(kT) (P_{ss}^{new} - P_{ss}^{min}) P_{ss}^{new} (t_D + 1/k) - \alpha V_{crit} [P_{ss}^{new} - P_{crush}(\gamma/\gamma_c)] \approx 0 \tag{52}$$

Rewriting this leads to the quadratic equation:

$$\begin{aligned} &a P_{ss}^{new\,2} - b P_{ss}^{new} + c = 0 \\ &a = 1 \\ &b = P_{ss}^{min} + \lambda \gamma / [\gamma_c (t_D + 1/k)] \\ &c = (\gamma/\gamma_c)^2 \lambda P_{crush} / (t_D + 1/k) \end{aligned} \tag{53}$$

Equation (40) is the solution of equation (53), where:

$$\lambda = \alpha V_{crit} \gamma_c kT / [\gamma N_0 (\gamma_c - \gamma) S] \tag{54}$$

Parameter values

The Yount article [3] reports the following parameter values:

Parameter	**Value**
γ	17.9 dyn/cm = 0.179 N/m
γ_c	257 dyn/cm = 2.57 N/m
τ_R	20160 min
r_0^{min}	0.80 µm
λ	7500 fsw min = 250 bar min

Some adaptations to VPM

In the Yount/Maiken/Baker article [4] VPM is applied to reverse dive profiles ($P_{ss} < P_{crush}$). Some adaptations are made to the VPM.

First, the descent is not assumed to be instantaneous but takes place at a certain rate. During descent gas is loaded into the tissue compartments, leading to a smaller P_{crush} value than on instantaneous descent (no gas loading). This effects the faster tissues more than the slower ones. Using the Schreiner equation one can derive a new, more general version of equation (39) for compartment j:

$$P_{ss}^{min}{}_j = 2\gamma(\gamma_c - \gamma) / \gamma_c r_0^{min}{}_j + \gamma/\gamma_c \Delta_j \tag{55}$$

The effects of nuclear regeneration have not been taken into account in this equation. In this equation the set of effective crushing pressures Δ_j is given by:

$$\Delta_j = P_{crush}(1 - Q_{N2}) + Q_{N2} R_c/k_j [1 - \exp(-k_j t_c)] \tag{56}$$

In this equation is Q_{N2} the Nitrogen fraction in the breathing gas mixture and R_c is the crushing change rate of the partial Nitrogen pressure. In the case of rapid descent, where in the limit t_c approaches 0, $R_c t_c$ approaches P_{crush} and Δ_j goes to P_{crush}. This results in the original equation (19).

In the Yount/Maiken/Baker article [4] equation (40) has been replaced by:

$$P_{ss}^{new} = 1/2 [b + (b^2 - 4c)^{1/2}] \tag{57}$$

where

$$b = P_{ss}{}^{min}{}_j + \lambda\, \gamma / [\gamma_c (t_D + 1/k_j)] - (P_{tj}{}^{dive} - P_m)t_D / [2 (t_D + 1/k_j)]$$

$$c = (\gamma/\gamma_c)^2\, \lambda\, P_{crush} / (t_D + 1/k_j) - P_{ss}{}^{min}{}_j\, (P_{tj}{}^{dive} - P_m)t_D / [2 (t_D + 1/k_j)]$$

In this equation $P_{tj}{}^{dive}$ denotes the set of compartment tissue tensions. The last terms have been added to b and c, compared to equation (40). These terms become zero for saturated, not-metabolizing systems, where $P_{tj}{}^{dive}$ is $P_t \approx P_m$.

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